JEE-Main-16-03-2021-Shift-2 (Memory Based)

PHYSICS

Question: For a damped oscillator, damping constant is 20 gm/s, mass is 500g. Find time taken for the amplitude to become half the initial?

Options:

(a) 50

(b) ln 2

(c) 50 ln2

(d) $\frac{25}{2}\ln 2$

Answer: (c)

Solution:

$$A = A_0 e^{-\frac{bt}{2m}}$$

$$\frac{A_0}{2} = A e^{-\frac{bt}{2m}}$$

$$2 = e^{\frac{bt}{2m}}$$

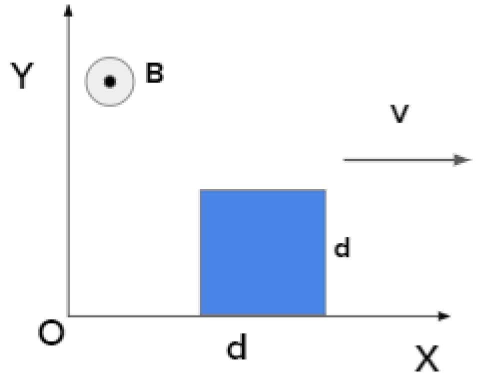
$$\ln 2 = \frac{bt}{2m}$$

$$t = \frac{2m}{b}\ln 2$$

$$= \frac{2\times 500}{20}\ln 2$$

$$t = 50\ln 2$$

Question: A square loop of side d is moved with velocity $v\hat{i}$ in a non-uniform magnetic field $\frac{B_0}{a}x\hat{k}$. Then the emf induced in the loop shown is?



Options:

(a) $\frac{B_0 dv}{a}$

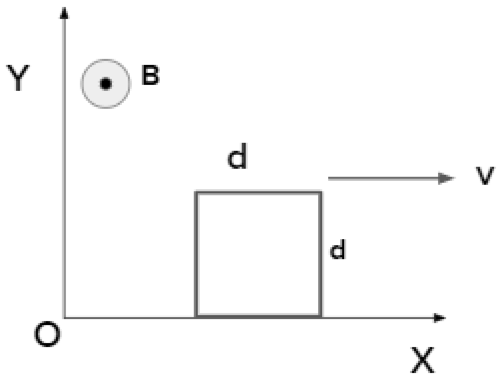
(b) $\frac{B_0 a^2 v}{d}$

(c) $2\frac{B_0 d^2 v}{a}$

(d) $\frac{B_0 d^2 v}{a}$

Answer: (d)

Solution:



e.m.f $=\frac{d\phi}{dt}$

$\phi = BA$

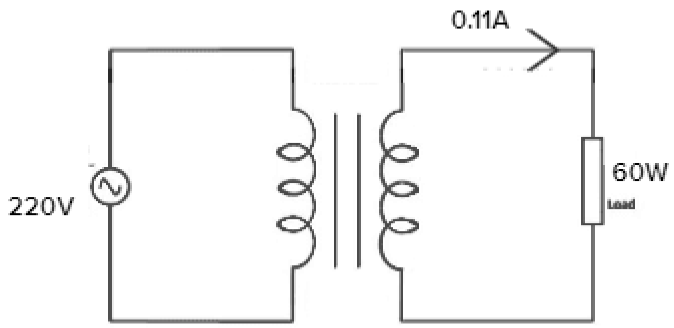
$\phi = \frac{B_0}{a} x \times d^2$

$$\frac{d\phi}{dt} = \frac{d\left(\frac{B_0 d^2}{a}.x\right)}{dt}$$

e.m.f $=\frac{B_0 d^2}{a}.\frac{dx}{dt}$

e.m.f. $=\frac{B_0 d^2}{a}.v$ $\left\{v = \frac{dx}{dt}\right\}$

Question: For the diagram shown, what is the type of transformer?



Options:
(a) Step-up
(b) Step-down
(c) Auxiliary
(d) Axial
Answer: (a)
Solution:

$V_{\text{input}} = 220V$

$P_{\text{output}} = 60W$

$P_{output} = V_{out} . I_{\text{input}}$

$60 = V_{out} \times 0.11$

$V_{out} = 545.45V$

$V_{out} > V_{in}$

Therefore, it is step-up transformer.

Question: If the range of single transmission between sending and receiving antennas of equal heights in 45 km. Then find the height of the antennas.
Options:
(a) 30 m
(b) 39.5 m
(c) 45 m
(d) 64 m
Answer: (b)
Solution:

Range $= \sqrt{2Rh_r} + \sqrt{2Rh_R}$

$R = 6.4 \times 10^6 m$ sss

$h_r = h_R = h$

Range $= 45 km$

$45 \times 10^3 = 2\sqrt{2 \times 6.4 \times 10^6 \times h}$

$\sqrt{h} = 6.2889$

$h = 39.55 m.$

Question: Find the resistance if it dissipates 10 mJ of energy per second when current of 1 mA passes through it.

Options:

(a) $1 k\Omega$

(b) $100 k\Omega$

(c) $10 k\Omega$

(d) $100 k\Omega$

Answer: (c)

Solution:

Given $P = 10\, mJ/s$

$P = 10 mW$

$I = 1 mA$

$P = I^2 R$

$10\, mW = (1 mA)^2 R$

$10 \times 10^{-3} = (1 \times 10^{-3})^2 R$

$10 \times 10^{-3} = 10^{-3} \times 10^{-3} \times R$

$R = 10^4$

$R = 10\, k\Omega$

Question: The focal length of a lens whose refractive index is same as that of the outside medium is?

Options:

(a) Zero

(b) Unity

(c) Infinity

(d) Can't be found

Answer: (c)

Solution:

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

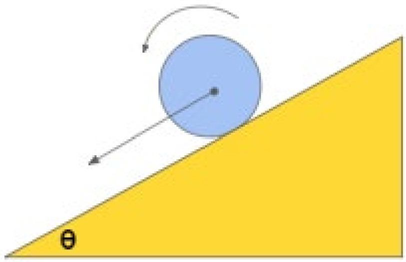
Refractive index of lens is same as medium

So, $\frac{\mu_2}{\mu_1} = 1$

$\frac{1}{f} = 0$

$\Rightarrow f = \infty$

Question: The acceleration of a disc rolling (purely) down an inclined plane of inclination θ is given as $a = \frac{xg \sin\theta}{3}$ Find x.



Answer: 2.00
Solution:
We know that for a body rolling down an inclined plane

$$a=\frac{g\sin\theta}{1+\frac{k^2}{R^2}}$$

For disc

$$mk^2=\frac{mR^2}{2}$$

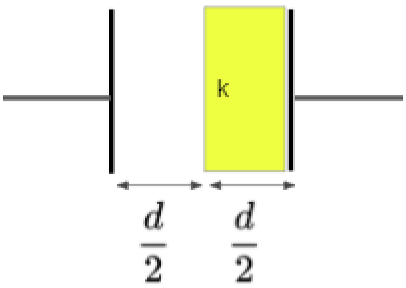
$$\Rightarrow k^2=\frac{R^2}{2}$$

$$a=\frac{g\sin\theta}{1+\frac{R^2}{2R^2}}$$

$$a=\frac{2}{3}g\sin\theta$$

So, $x=2$

Question: Find the equivalent capacitance for the given figure
$A=0.2m^2, d-1m, k=3.2$



Options:
(a) $0.1\in_0$
(b) $0.2\in_0$
(c) $0.3\in_0$
(d) $0.4\in_0$
Answer: (c)
Solution:
We can consider the shown capacitor as series combination of two capacitors.

$$C_1=\frac{2\in_0 A}{d} \text{ and } C_2=\frac{2K\in_0 A}{d}$$

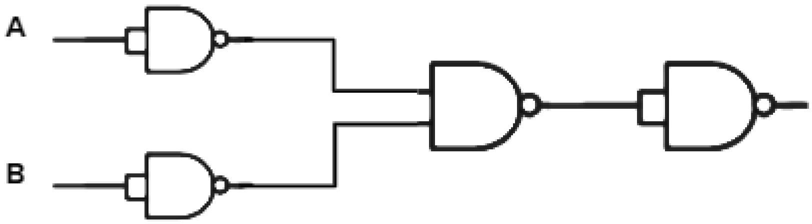
$$C_{eq}=\frac{C_1C_2}{C_1+C_2}$$

$$=\frac{\frac{2\in_0 A}{d}\times\frac{2K\in_0 A}{d}}{\frac{2\in_0 A}{d}(1+K)}$$

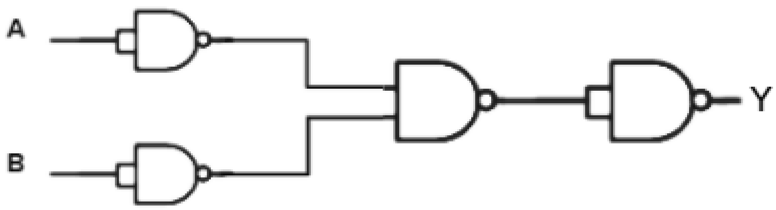
$$=\frac{2\in_0\times3.2\times0.2}{(1+3.2)}$$

$$C_{eq}=0.304\in_0\approx0.3\in_0$$

Question: This is equivalent to:



Options:
(a) OR
(b) AND
(c) NOR
(d) NAND
Answer: (c)
Solution:



$$Y=\overline{\overline{A.A}.\overline{B.B}}$$

$$=\overline{\overline{(A+A).(B+B)}}$$

$$=\overline{\overline{A.B}}$$

$$=\overline{A+B}$$

Hence it’s a NOR Gate.

Question: There are two species A & B with half lives 54 & 18 minutes respectively. The time after which concentration of A is 16 times that of B will be -
Options:
(a) 27 min

(b) 54 min
(c) 81 min
(d) 108 min
Answer: (d)
Solution:
$t_A = 54\,\text{min}$

$t_B = 18\,\text{min}$

$N_A = 16\,N_B$

$N_0 e^{-\lambda_A t} = 16 N_0 e^{-\lambda_B t}$

$e^{(\lambda_B - \lambda_A)t} = 16$

$e^{\left(\frac{t}{t_B} - \frac{t}{t_A}\right)\ln 2} = 16$

$2^{\left(\frac{t}{t_B} - \frac{t}{t_A}\right)} = 2^4$

$t(\frac{1}{18} - \frac{1}{54}) = 4$

$t = 108\,\text{min}$

Question: If half life of an element is 20 minutes. Find the time interval of 33.33% and 66.66% decay.
Options:
(a) 10 minutes
(b) 20 minutes
(c) 40 minutes
(d) 80 minutes
Answer: (b)
Solution:
The relation between decay constant (λ) and half-life (T) is:

$$\lambda = \frac{\log 2}{T_{\frac{1}{2}}} = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{20} = 0.03465\,\text{per min}$$

Time of decay, $t = \dfrac{2.303}{\lambda}\log_{10}\dfrac{N_0}{N}$

Time of decay for 33.33% disintegration is:

$$t_1 = \frac{2.303}{0.03465}\log_{10}\frac{100}{66.66} = 11.71\,\text{min}$$

Time of decay for 66.66 % disintegration is:

$$t_2 = \frac{1.303}{0.03465}\log_{10}\frac{100}{33.33} = 31.71\,\text{min}$$

Hence, difference of time is:

$\Delta t = t_2 - t_1 = 31.71 - 11.71 = 20\,\text{min}$

Question: Red and violet light have -

Options:

(a) Same frequency, different wavelength

(b) Different frequency, same wavelength

(c) Different frequency, different wavelength

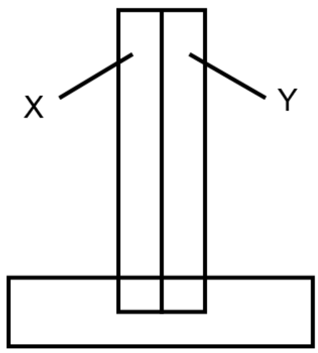
(d) Same frequency, same wavelength

Answer: (c)

Solution:

Violet light has a higher frequency and shorter wavelength than red light.

Question: A bimetallic strip consists of metals X and Y. it is mounted rigidly at the base as shown. The metal X has a higher coefficient of expansion compared to that for metal Y. when the bimetallic strip is placed in a cold bath:



Options:

(a) The combination will bend with X on convex side.

(b) The combination will bend with Y on convex side

(c) There will be no bending

(d) Cannot be predicted

Answer: (b)

Solution:

As coefficient of thermal expansion of X is more. On cooling, it will shrink more. So the strip will bend with Y on convex side.

Question: An electron and a proton are accelerated by same voltage difference. Find the ratio of the de Broglie wavelength of electron: Proton. $\left(m_p : m_e = 1860 : 1\right)$

Options:

(a) $\frac{1860}{1}$

(b) $\frac{41.4}{1}$

(c) $\frac{43}{1}$

(d) $\frac{4}{1}$

Answer: (c)

Solution:

For electron

$$\lambda_e = \frac{12.27}{\sqrt{V}}\overset{\circ}{A} \quad ..(1)$$

For proton

$$\lambda_p = \frac{0.286}{\sqrt{V}}\overset{\circ}{A} \quad ..(2)$$

So, from eq. (1) and (2)

$$\left[\frac{\lambda_e}{\lambda_P} = \frac{\left(\frac{12.27}{\sqrt{V}}\right)}{\frac{0.286}{\left(\sqrt{V}\right)}} = \frac{42.90}{1} or \frac{43}{1}\right]$$

Question: A charge 'q' moves by a distance $'dl'$ under the presence of magnetic field 'B'. Find the work done by the field?

Options:

(a) $q\overline{B}.\overline{dl}$

(b) $\frac{q^2\overline{B}.\overline{dl}}{2}$

(c) ∞

(d) Zero

Answer: (d)

Solution:

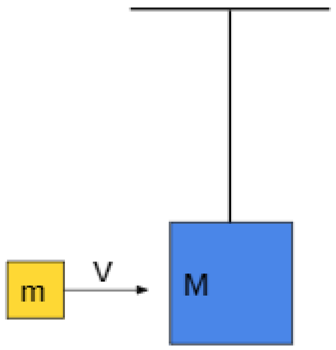
The magnetic force acts in such a way that the direction of the magnetic force and velocity and always perpendicular to each other. If force and velocity are perpendicular, then force F displacement will also be perpendicular

So, $W = F.d\cos\theta$

If $\theta = 90°$

$\boxed{W = 0}$

Question: A block of mass = 5.99 kg hangs from string. A small mass m = 10 grams strikes it with velocity v. if the height to which system rises is 9.8 cm, then find v. Assume perfectly inelastic collision and $g = 10\,m/s^2$.



Options:
(a) 800 m/s
(b) 840 m/s
(c) 900 m/s
(d) 1000 m/s
Answer: (b)
Solution:
By law of momentum conservation

$$mV + 0 = (m+M)V'$$

$$0.01 \times V = (0.01 + 5.99)V'$$

$$V' = \frac{0.01V}{6} = \frac{V}{600} m/s \quad ....(1)$$

By energy conservation,

$$\frac{1}{2}(m+M)V'^2 = (m+M)gh$$

From eq. (1)

$$\frac{1}{2}(6)\left(\frac{V}{600}\right)^2 = (6) \times 10 \times \frac{98 \times 10^{-2}}{10}$$

$$V^2 = 705600$$

$$V = 840\, m/s$$

Question: 500 Joules of heat is dissipated when 1.5 Amperes of current is passed through a resistor for 20 seconds. If current is changed to 3 A, then how much heat will be dissipated in same time.
Options:
(a) 500 Joules
(b) 125 Joules
(c) 2000 Joules
(d) 1000 Joules
Answer: (c)
Solution:

$$H_1 = i_1^2 Rt$$

$H_1 = (1.5)^2 \times R \times 2 = 500$

$R = \frac{500}{20 \times (1.5)^2} = \frac{500}{20 \times 2.25} \quad ...(1)$

So for, $i = 3Amp$

$H_2 = i_2^2 Rt$

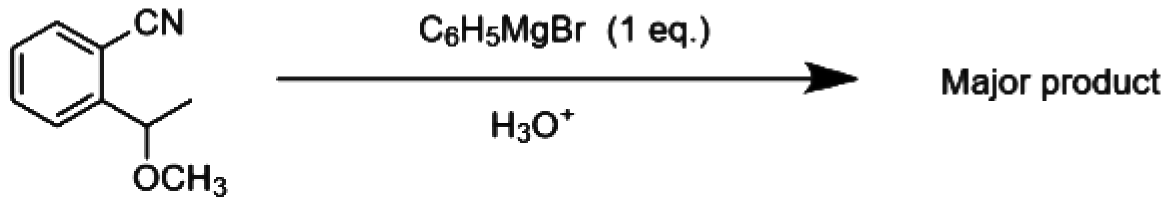
From eq (1) $H_2 = (3)^2 \times \left(\frac{500}{20 \times 2.25} \right) \times 20$

$H_2 = 2000\,\text{Joules}$

JEE-Main-16-03-2021-Shift-2 (Memory Based)

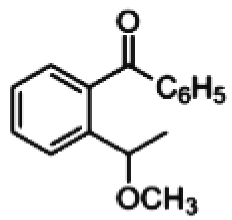
CHEMISTRY

Question:

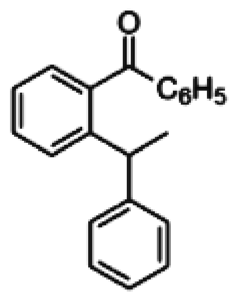


Options:

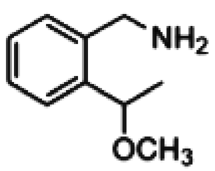
(a)



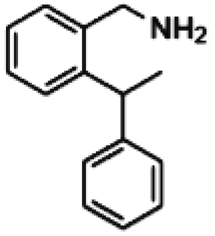
(b)



(c)

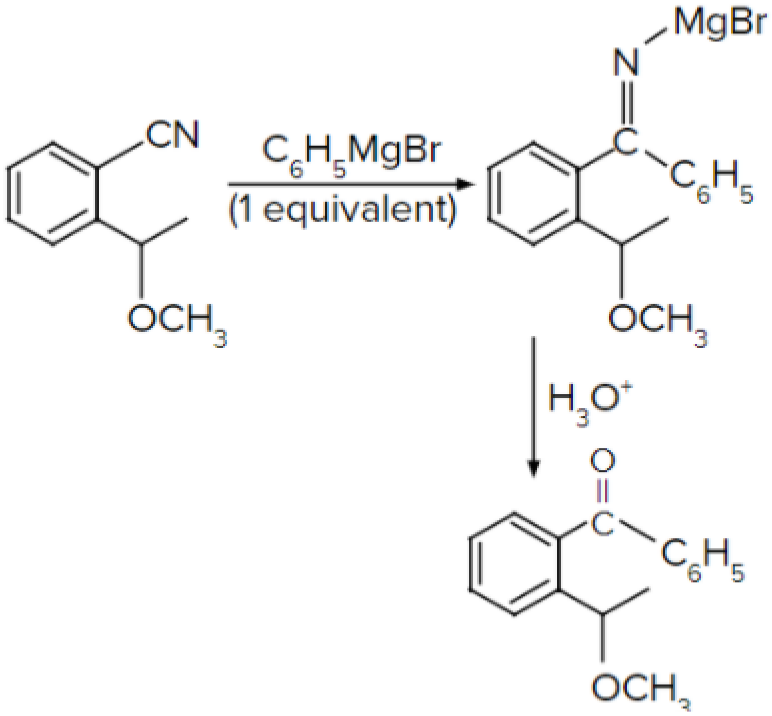


(d)



Answer: (a)

Solution:



Question: Wood laminates are made up of:
Options:

(a) Polystyrene

(b) PVC

(c) Urea-formaldehyde resins

(d) Bakelite

Answer: (c)

Solution:

Urea-formaldehyde resins

I) Urea, II) Formaldehyde

$\left(NH-CO-NH-CH_2 \right)_n$

For making unbreakable cups and laminated sheets

Question: The number of orbitals that can be formed with n = 5, l = 4, m_l = +2
Options:

(a) 1

(b) 2

(c) 3

(d) 4

Answer: (a)

Solution:

n = 5, l = 4

5g

–4, –3, –2, –1, 0, +1, +2, +3,

Question: The constituents of greenhouse gases:
I. CO_2, **II**. H_2O
III. CH_4, **IV.** O_3
Options:

(a) Only I

(b) I and II

(c) I, II, III

(d) All of these

Answer: (d)

Solution: The main constituents of Green-house gases are methane, Carbon dioxide, ozone, nitrous oxide and water vapours.

Question: Which of the following cannot be reduced by coke?
Options:

(a) FeO
(b) Al_2O_3
(c) CaO

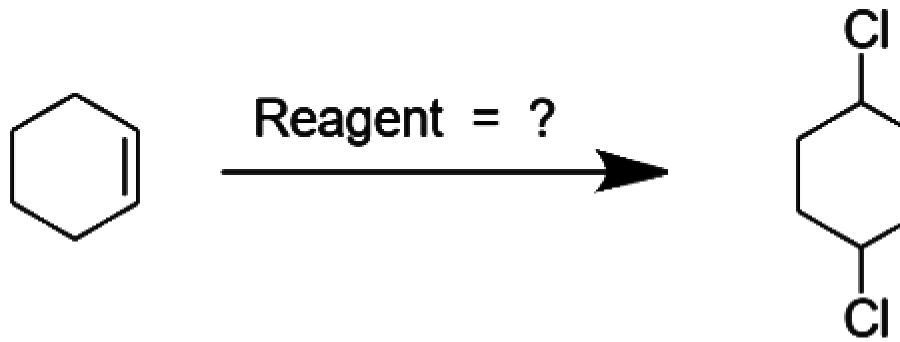
(d) Cu_2O

Answer: (c)

Solution: Oxides of strong electropositive metals such as K, Ca, Na, Al and Mg are very stable

It is difficult to reduce them into metallic state by carbon reduction

Question:



Options:

(a) Zn-Hg

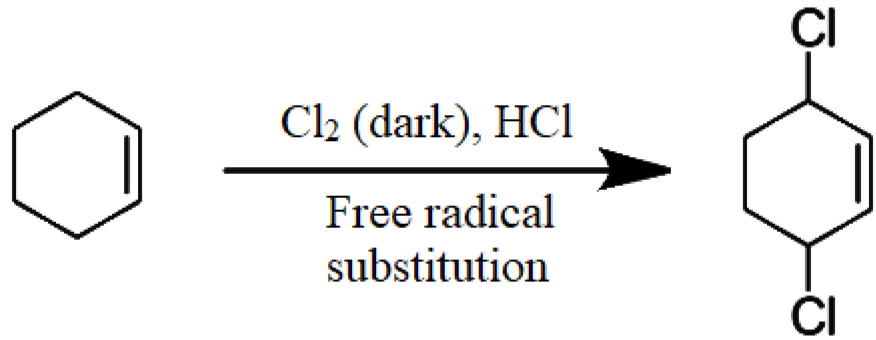
(b) HCl, Anhydrous $AlCl_3$

(c) Cl_2 (dark), HCl

(d) Cl_2, $FeCl_3$

Answer: (c)

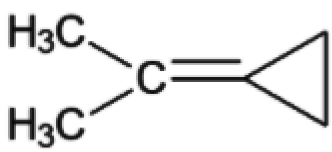
Solution:



Question: Ozonolysis of X gives A which is an aldehyde. A on heating with silver oxide gives beautiful silver mirror lining. X is ?

Options:

(a)

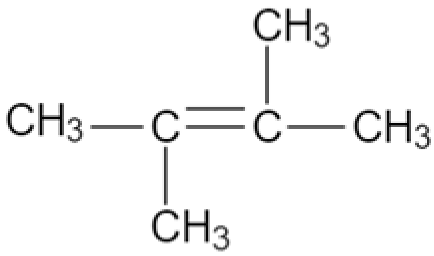


(b)

$HC\equiv C-CH_3$

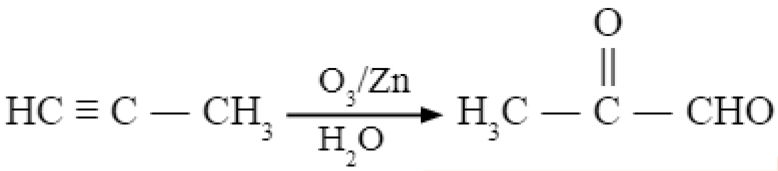
(c)

(d)



Answer: (b)

Solution:



Question: S1: Sodium hydride can be used as an oxidizing agent.

S2: Pyridine is base because of lone pair.

Options:

(a) S_1 is correct

(b) S_2 is incorrect

(c) Both are correct

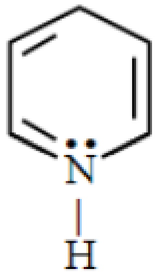
(d) Both are not correct

Answer: (b)

Solution:

1) NaH is reducing agent

2)



Pyridine is base

Question: The secondary group proteins have which of the following forces
Options:

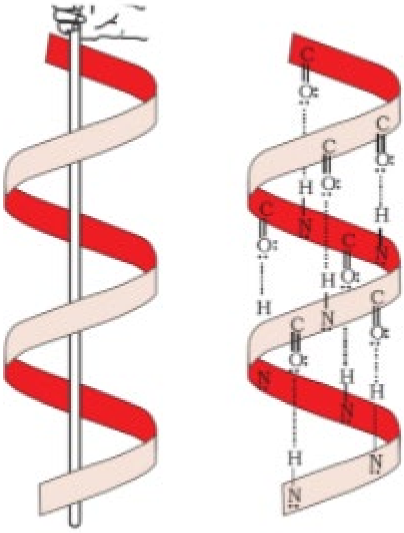
(a) Vander waals forces
(b) Hydrogen bond

(c) Covalent bond

(d) Ionic bond

Answer: (b)

Solution:



Secondary groups of proteins are produced maintained by H-bonding. Two types of secondary structure

i.e., α – Helix and β -pleated sheet

Question: Role of NaOH in ammonolysis of halide?
Options:

(a) Stabilizes the transition state

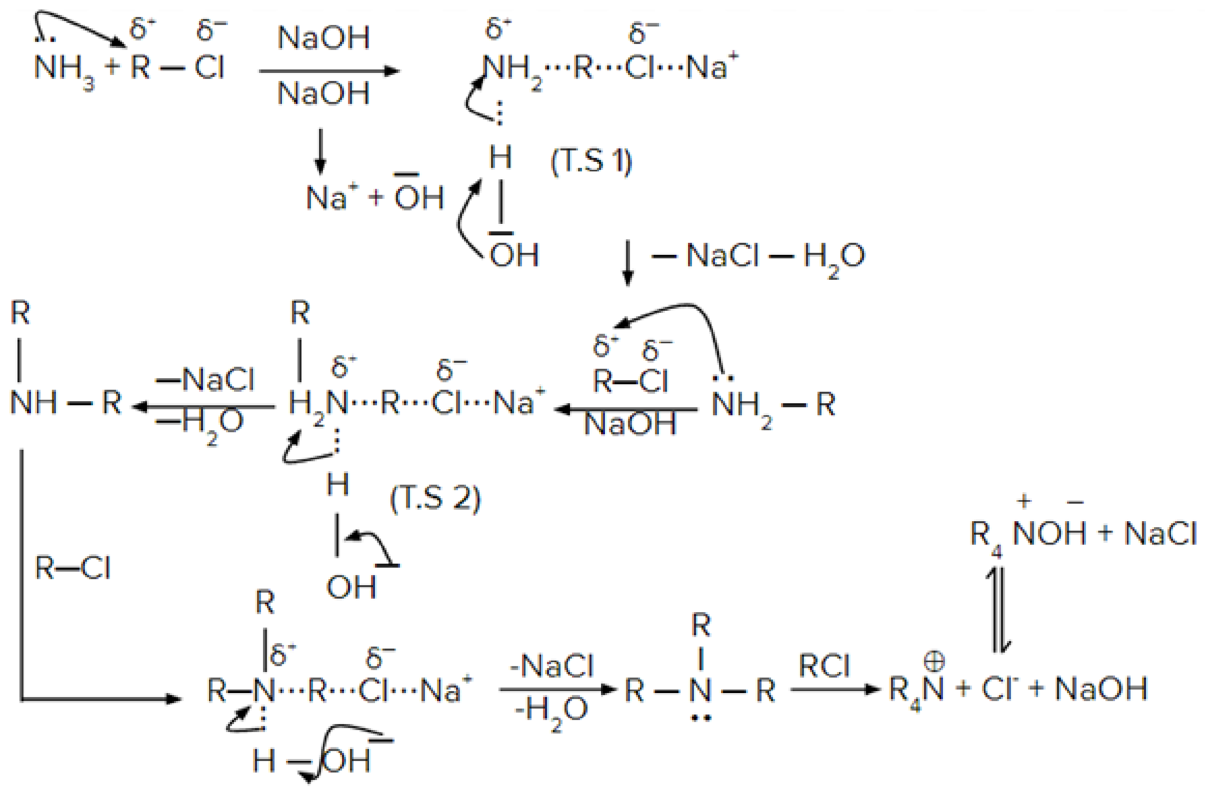
(b) Consumes the leaving group

(c) Both a and b

(d) None of these

Answer: (c)

Solution:



Question:

$Pb(NO_3)_2 + Cr_2(SO_4)_7 \rightarrow PbSO_4$
$Pb(NO_3)_2$ = 35 ml, 0.15 M
$Cr_2(SO_4)_7$ = 20 ml, 0.12 M
Find the moles of $PbSO_4$
Options:

(a) 5.25×10^{-3} moles

(b) 3.25×10^{-3} moles

(c) 1.25×10^{-3} moles

(d) 2×10^{-3} moles

Answer: (a)

Solution:

$$3Pb(NO_3)_2 + 3Cr_2(SO_4)_3 \rightarrow 3PbSO_4 + 2Cr(NO_3)_3$$

millimole = 5.25 2.4

Here L.R is $Pb(NO_3)_2$

Moles of $PbSO_4$ formed = 5.25 millimoles = 5.25×10^{-3} moles

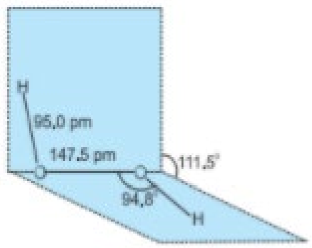
Question: Which of the following is incorrect statement regarding H_2O_2?
Options:

(a) It is used both as oxidising agent and reducing agent
(b) It is used in effluents
(c) Both hydroxyl groups are present in the same plane
(d) Its shape is open book type structure

Answer: (c)

Solution: Hydrogen peroxide is an important chemical used in pollution control treatment of domestic and industrial effluents

Hydrogen peroxide has a non-planar structure. The molecular dimensions in the gas phase and solid phase are shown in figure



Question: The volume of 1 M NaOH required for complete neutralization of 100 ml of 1 M of H_3PO_3 and 100 ml of 2 M H_3PO_4 is:

Options:

(a) 200 ml, 200 ml

(b) 200 ml, 400 ml

(c) 200 ml, 600 ml

(d) 200 ml, 800 ml
Answer: (c)

Solution:

Eq of NaOH = eq of H_3PO_3

$= 0.1 \times 1 \times 2$

$V \times 1 = 0.2$

$V = 0.2$ litre = 200 ml

Eq of NaOH = eq of H_3PO_4

$= 0.1 \times 2 \times 3$

$V \times 1 = 0.6$ litre

$V = 600$ ml

Question: Which halogen cannot form FeX_3 and FeX_2?

Options:

(a) I

(b) Br

(c) F

(d) Cl

Answer: (a)

Solution: FeX_3 and FeX_2 is unstable

FeI_3 does not exist because Fe^{3+} oxidises I^- to I_2

Question: Atomic number of X, Y and Z are 33, 53, an 83 respectively, then:

Options:

(a) X and Z are non-metals and Y is metal

(b) X is metalloid, Y is non-metal and Z is metal

(c) X and Z are metals, Y is non-metal

(d) None of these

Answer: (b)

Solution:

X = Arsenic

Y = Iodine

Z = Bismuth

Question: If half-life of an element is 20 minutes. Find the time interval of 33% decay and 67% decay

Options:

(a) 13.05

(b) 23.45

(c) 33.25

(d) 41.15

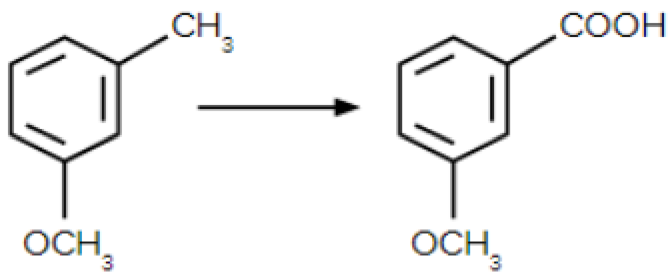
Answer: (a)

Solution:

$$t_{1/2} = 20\,\text{min}$$
$$K = \frac{0.613}{20} = 0.03$$

$$t_{67\%} = \frac{2.303}{0.03}\log\left(\frac{100}{33}\right)$$
$$= \frac{2.303}{0.03}\log(3.03)$$
$$= \frac{2.303}{0.03}\times 0.48 = 36.84$$
$$t_{33\%} = \frac{2.303}{0.03}\log\left(\frac{100}{67}\right)$$
$$= \frac{2.303}{0.03}\log(1.5)$$
$$= \frac{2.303}{0.03}\times 0.17 = 13.05$$

Question: The conversion is carried out



Options:

(a) $NaBH_4$

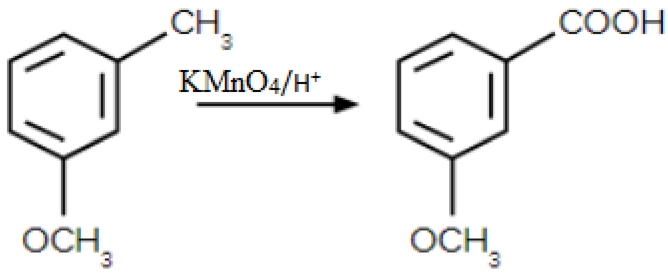
(b) $KMnO_4/H^+$

(c) $LiAlH_4$

(d) H_2O/H^+

Answer: (b)

Solution:



Question: Match the following.

Tests/ reagents (Column I)	**Tests/ reagents (Column II)**
(A) Lassaigne's test	**(i)** Carbon
(B) CuO	**(ii)** N, P, S and halogen
(C) Silver nitrate	**(iii)** Halogen only
(D) Sodium Nitroprusside	**(iv)** Sulphur

Options:

(a) (A) → (ii); (B) → (i); (C) → (iii); (D) → (iv)

(b) (A) → (iii); (B) → (ii); (C) → (i); (D) → (iv)

(c) (A) → (iv); (B) → (iii); (C) → (ii); (D) → (i)

(d) (A) → (i); (B) → (iii); (C) → (iv); (D) → (ii)

Answer: (a)

Solution:

Lassaigne's test ⇒ N, P, S and halogen

CuO ⇒ Carbon

Silver nitrate ⇒ Halogen only

Sodium Nitroprusside ⇒ Sulphur

JEE-Main-16-03-2021-Shift-2 (Memory Based)

MATHEMATICS

Question: $F(x+1) = xF(x)$ and $g(x) = \ln \mathrm{F}(x)$ Find $|g''(5) - g''(1)|$

Options:

(a)

(b)

(c)

(d)

Answer: ()

Solution:

$f(x+1) = xf(x)$

$f(x+N) = (x+N-1)f(x+N-1)$

$= (x+N-1)(x+N-2)f(x+N-2).....$

$f(x+N) = (x+N-1)(x+N-2)....(x-1)(x)f(x)$

$g(x+N) = \ln f(x+N) = \ln(x+N-1) + \ln(x+N-2) + ...\ln f(x)$

$\therefore g'(x+N) = \frac{1}{x+N-1} + \frac{1}{x+N-2} + ...\frac{1}{x} + g'(x)$

$g''(x+N) - g''(x) = \frac{-1}{(x+N-1)^2} - \frac{1}{(x+N-2)^2} .....\frac{-1}{x^2}$

Put $x = 1$ and $N = 4$

$g''(5) - g''(1) = -\left[\frac{1}{4^2} + \frac{1}{3^2} + \frac{1}{2^2} + \frac{1}{1^2}\right]$

$|g''(5) - g''(1)| = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} = \frac{205}{144}$

Question: C_1 and C_2 are two curves intersecting at $(1,1)C_1$ satisfy $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ and C_2 satisfy $\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$ Then area bounded by these two curves is

Options:

(a)

(b)
(c)
(d)
Answer: ()
Solution:

$$C_1 : \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v} \Rightarrow x\frac{dv}{dx} = \frac{-\left(1+v^2\right)}{2v}$$

$$\int\left(\frac{2v}{1+v^2}\right)dv = -\int\frac{dx}{x} \Rightarrow \ln\left(1+v^2\right)x = c$$

$$C_1 : \frac{\left(x^2 + y^2\right)}{x} = c = 2 \Rightarrow C_1 : x^2 + y^2 = 2x$$

$$C_2 : \frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

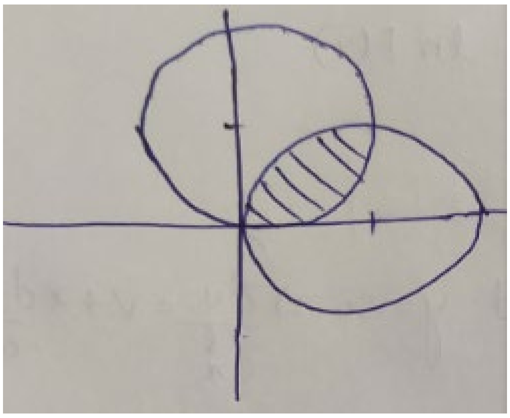
$$v + x\frac{dv}{dx} = \frac{2v}{1-v^2} \Rightarrow x\frac{dv}{dx} = \frac{v+v^3}{1-v^2}$$

$$\frac{1-v^2}{v\left(1+v^2\right)}dv = \frac{dx}{x} \Rightarrow \int\left(\frac{1}{v} - \frac{2v}{1+v^2}\right)dv = \int\frac{dx}{x}$$

$$\Rightarrow \ln\left(\frac{v}{1+v^2}\right) = \ln x + c \Rightarrow \frac{y}{x^2+y^2} = \frac{1}{2}$$

$$C_2 : x^2 + y^2 = 2y$$

$\therefore$ Area bounded between $\left(x-1\right)^2 + y^2 = 1$ and $x^2 + \left(y-1\right)^2 = 1$ is



Area = $\int_0^1 \left(\sqrt{2x-x^2}-\sqrt{1-x^2}-1\right)dx$

$$=\left[\frac{(x-1)}{2}\sqrt{2x-x^2}-\frac{1}{2}\sin^{-1}(x-1)-\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x-x\right]_0^1$$

$$=\left(\frac{\pi}{4}-1\right)-\left(\frac{-\pi}{4}\right)=\frac{\pi}{2}-1$$

Question: A six digit number is formed by the numbers 0, 1, 2, 3, 4, 5, 6 without repetition. Then the probability that the number is divisible by 3 is

Options:

(a) $\frac{11}{24}$

(b) $\frac{3}{7}$

(c) $\frac{4}{9}$

(d) $\frac{9}{56}$

Answer: (c)

Solution:

Given numbers are 0, 1, 2, 3, 4, 5, 6

Total number of 6-digit number = $6\times 6! = 720\times 6$

6-digit number divisible by 3

(a) when '0' is excluded = $6! = 720$

(b) when '0' is included = $2\times 5\times 5! = 1200$

$\therefore$ Required probability = $\frac{1920}{4320}=\frac{4}{9}$

Question: Let 'c' be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x.$ Then the equation of tangent to 'c' at $p(2,1)$ is:

Options:

(a) $x+3y=5$

(b) $x+2y=4$

(c) $x-y=1$

(d) $2x+y=5$

Answer: (c)

Solution:

Any point on parabola $y^2 = 4x$ is $(t^2, 2t)$

Mirror image of $(t^2, 2t)$ w.r.t $y = x$ is $(2t, t^2)$

$\therefore$ locus of 'C' is $x^2 = 4y$

$\therefore$ Equation of tangent to 'C' is at (2, 1) is $2x = 2(y+1)$ or $x = y+1$

Question: The maximum value of $f(x) = \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in R$

Options

(a) $\sqrt{5}$

(b) 5

(c) $\frac{3}{4}$

(d) $\sqrt{7}$

Answer: (a)

Solution:

$$f(x) = \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$$

$$= \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1 & -1 & 0 \\ 0 & -1 & \sin 2x - \cos 2x \end{vmatrix}$$

$\therefore f(x) = -\cos 2x + (\sin 2x - \cos 2x)(-\sin^2 x - 1 - \cos^2 x)$

$f(x) = \cos 2x - 2\sin 2x$

$\therefore$ Maximum value of $f(x) = \sqrt{1^2 + 2^2} = \sqrt{5}$

Question: If the points of intersections of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = 4b, b > 4$ lie on the curve $y^2 = 3x^2$ then 'b' is equal to

Options

(a) 12

(b) 6

(c) 10

(d) 5

Answer: (a)

Solution:

$\frac{x^2}{16} + \frac{y^2}{b^2} = 1$

$x^2 + y^2 = 4b$

$y^2 = 3x^2$

$\Rightarrow b = x^2$

$y^2 = 3b$

$\therefore \frac{b}{16} + \frac{3}{b} = 1$

$\Rightarrow b^2 - 16b + 48 = 0$

$\Rightarrow b = 4, 12$

$\because b > 4 \Rightarrow b = 12$

Question: $f(x) = \begin{cases} \frac{\cos^{-1}\left(1 - \{x\}^2\right).\sin^{-1}\left(1 - \{x\}\right)}{\{x\}\left(1 - \{x\}\right)\left(1 + \{x\}\right)}; & x \neq 0 \\ \alpha; & x = 0 \end{cases}$, Find α if $f(x)$ is continuous

Options:

(a)

(b)

(c)

(d)

Answer: ()

Solution:

$\because f\left(x\right)$ is continuous

$\therefore f\left(0\right)=\lim\limits_{x\to 0} f\left(x\right)$

$$\Rightarrow \alpha=\lim_{x\to 0}\frac{\cos^{-1}\left(1-\{x\}^2\right)\cdot\sin^{-1}\left(1-\{x\}\right)}{\{x\}\left(1-\{x\}\right)\left(1+\{x\}\right)}$$

$$=\frac{\pi}{2}\lim_{x\to 0}\frac{\cos^{-1}\left(1-x^2\right)}{x}$$

$$=\frac{\pi}{2}\lim_{x\to 0}\frac{-1\left(-2x\right)}{\sqrt{1-\left(1-x^2\right)^2}}$$

$$=\frac{\pi}{2}\lim_{x\to 0}\frac{2x}{x\sqrt{2-x^2}}=\frac{\pi}{\sqrt{2}}$$

Question: $\int\limits_0^{10}\frac{[x]e^{[x]}}{e^{x-1}}dx$

Options:

(a) $9\left(e-1\right)$

(b) $9\left(e+1\right)$

(c) $45\left(e-1\right)$

(d) $45\left(e+1\right)$

Answer: (c)

Solution:

$$I=e\int\limits_0^{10}\frac{[x]e^{[x]}}{e^x}dx$$

$$= e\left[e\int_1^2 e^{-x}dx + 2e^2\int_2^3 e^{-x}dx + ...9e^9\int_9^{10} e^{-x}dx\right]$$

$$= -e\left[e\left(e^{-2}-e^{-1}\right)+2e^2\left(e^{-3}-e^{-2}\right)+3e^3\left(e^{-4}-e^{-3}\right)+...9e^9\left(e^{-10}-e^{-9}\right)\right]$$

$$= -e\left[\left(e^{-1}-1\right)+2\left(e^{-1}-1\right)+3\left(e^{-1}-1\right)+...9\left(e^{-1}-1\right)\right]$$

$$= -e\left[45e^{-1}-45\right] = 45\left[e-1\right]$$

Question: ABCD is a rectangle with 5, 6, 7, 9 points on side AB, CD, BC and AD respectively. Let α be the number of quadrilateral that can be formed using these points with vertices and different sides and let β be the number of triangles formed with vertices on different side. Then what is $\alpha - \beta$?

Options:

(a) 1890

(b) 1173

(c) 717

(d) 819

Answer: (c)

$$\alpha = {}^5C_1 \times {}^6C_1 \times {}^7C_1 + {}^6C_1 \times {}^7C_1 \times {}^9C_1 \times {}^5C_1 + {}^9C_1 \times {}^5C_1 \times {}^6C_1$$

$$= 210 + 378 + 315 + 270 = 1173$$

$$\beta = {}^5C_1 \times {}^6C_1 \times {}^7C_1 \times {}^9C_1 = 1890$$

$$\therefore \beta - \alpha = 717$$

Question: x, y, z be a point on plane passing through (42, 0, 0), (0, 42, 0) and (0, 0, 42) then find the value of:

$$\frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2} + \frac{z-12}{(x-11)^2(y-19)^2} + 3 - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

Answer: 3.00

Solution:

Equation of plane is $x + y + z = 42$

$$\Rightarrow (x-11)+(y-19)+(z-12) = 0$$

Let $x - 11 = 4;\ y - 19 = v;\ z - 12 = w$

i.e. $u + v + w = 0$

$$\therefore \frac{u}{v^2 \cdot w^2}+\frac{v}{u^2 \cdot w^2}+\frac{w}{u^2 \cdot v^2}-\frac{3}{u \cdot v \cdot w}+3$$

$$=\left(\frac{u^3+v^3+w^3-3uvw}{u^2v^2 \cdot w^2}\right)+3=3$$

$$\left[\because u+v+w=0 \Rightarrow u^3+v^3+w^3=3uvw\right]$$

Question: $2^{\frac{(|z|+3)(|z|-1)}{|z|+1}} \geq \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|$ Find the minimum value of $|z|$.

Answer: 3.00

Solution:

$$2^{\frac{(|z|+3)(|z|-1)}{|z|+1}} \geq 2\log_2\left|5\sqrt{7}+9i\right|$$

$$\because \left|5\sqrt{7}+9i\right|=\sqrt{25\times 7+81}=\sqrt{256}=16$$

$$\therefore 2\log_2 16=2\log_2 2^4=8=2^3$$

$$\Rightarrow \frac{(|z|+3)(|z|-1)}{|z|+1} \geq 3 \Rightarrow |z|^2+2|z|-3 \geq 3|z|+3$$

$$|z|^2-|z|-6 \geq 0 \Rightarrow (|z|-3)(|z|+2) \geq 0$$

$$|z| \geq 3$$

$$|z|_{\min}=3$$

Question: $\frac{1}{16}, a, b$ are G.P, $\frac{1}{a}, \frac{1}{b}, 6$ are in A.P Then find the value of $72(a+b)$

Answer: 54.00

Solution:

$$a^2=\frac{b}{16}; \frac{2}{b}=\frac{1}{a}+6 \Rightarrow b=\frac{2a}{1+6a}$$

$$\therefore 16a^2=\frac{2a}{1+6a} \Rightarrow 96a^2+16a-2=0$$

$$48a^2+8a-1=0 \Rightarrow 48a^2+12a-4a-1=0$$

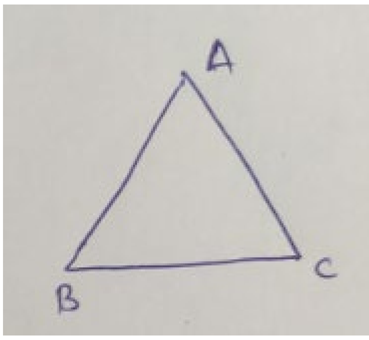
$$12a(4a+1)-(4a+1)=0 \Rightarrow a=\frac{-1}{4},\frac{1}{12} \Rightarrow b=1,\frac{1}{9}$$

$$\therefore 72(a+b)=54 \text{ or } 14$$

Question: Two sides of ΔABC are 5 and 12. Area of ΔABC is 30. Find $2R+r,$ where R is circumradius and r is inradius.

Answer: 15.00

Solution:



Let $a=5,\ b=12,\ \Delta=30$

$$\Delta=\frac{1}{2}\cdot a\cdot b\cdot \sin c=\frac{1}{2}\times 5\times 12\cdot \sin c=30$$

$$\therefore \sin c=1 \Rightarrow c=90 \Rightarrow c=13$$

$$\Rightarrow 2R=\frac{c}{\sin c}=13;\ r=\frac{\Delta}{s}=\frac{30}{15}=2$$

$$\therefore 2R+r=13+2=15$$

Question: $A=XB,$ A and B are 2 × 1 matrices

$$A=\begin{bmatrix}a_1\\a_2\end{bmatrix},\ B=\begin{bmatrix}b_1\\b_2\end{bmatrix},\ X=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & -1\\1 & k\end{bmatrix}, A=XB\ ,\ a_1^2+a_2^2=\frac{2}{3}\left(b_1^2+b_2^2\right).$$ Find k.

Answer: 1.00

Solution:

$$A=\begin{bmatrix}a_1\\a_2\end{bmatrix},\ B=\begin{bmatrix}b_1\\b_2\end{bmatrix},\ X=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & -1\\1 & k\end{bmatrix}$$

$$A=XB$$

$$\begin{bmatrix}a_1\\a_2\end{bmatrix}=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & -1\\1 & k\end{bmatrix}\begin{bmatrix}b_1\\b_2\end{bmatrix}=\frac{1}{\sqrt{3}}\begin{bmatrix}b_1-b_2\\b_1+kb_2\end{bmatrix}$$

$$\therefore a_1 = \frac{b_1 - b_2}{\sqrt{3}};\ a_2 = \frac{b_1 + kb_2}{\sqrt{3}}$$

$$a_1^2 + a_2^2 = \frac{b_1^2 + b_2^2 - 2b_1b_2 + b_1^2 + k^2b_2^2 + 2kb_1b_2}{3} = \frac{\left(b_1^2 + b_2^2\right)}{3}$$

$$b_2^2 = kb_2^2 + 2b_1b_2\left(k-1\right)$$

$$\Rightarrow k = 1$$

Question: $A = \{2,3,4,..30\}$, (a, b) and (c, d) are equivalent of $ad = bc$ then number of elements equivalent to (4, 3)

Answer: 7.00

Solution:

$$A = \{2,3,4,..30\};\ \frac{a}{b} = \frac{c}{d}$$

$$\therefore (4,3) = \frac{4}{3} = \frac{8}{6} = \frac{12}{9} = \frac{16}{12} = \frac{20}{15} = \frac{24}{18} = \frac{28}{21}$$

$\therefore$ Total number of elements = 7

Question: $\sum_{k=0}^{n}(-1)^k\, {}^nC_k\left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \ldots\left(\frac{31}{32}\right)^k\right]$, $63A = 1 - \frac{1}{2^{30}}$ Find n

Answer: 6.00

Solution:

$$A = \sum_{k=0}^{n}(-1)^k\, {}^nC_k\left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \ldots\left(\frac{31}{32}\right)^k\right]$$

$$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n + \left(\frac{1}{8}\right)^n + \left(\frac{1}{16}\right)^n + \left(\frac{1}{32}\right)^n$$

$$A = \frac{\left(\frac{1}{2}\right)^n\left[1 - \left(\frac{1}{2}\right)^{5n}\right]}{1 - \left(\frac{1}{2}\right)^n}$$

When $n = 6 \Rightarrow 63A - 1 = \frac{1}{2^{30}} \Rightarrow n = 6$